# Relative importance of the epistatic components of genotypic variance in non-inbred populations 

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#### Abstract

There are theoretical approaches about genic interaction in polygenic systems and methodologies to confirm its occurrence. However, some relevant aspects as the relative importance of the epistatic components of genotypic variance deserve further investigation. Considering complementary, duplicate, recessive, dominant, dominant and recessive epistasis, duplicate genes with cumulative effects and non-epistatic genic interaction, relative magnitudes of the epistatic components of genotypic variance assuming digenic epistasis were analyzed. Regardless of the type of epistasis and gene frequencies, the magnitudes of the epistatic components are proportional to the complexity of the polygenic system, the number of interacting genes and the magnitude of the epistatic effects relative to the deviations a (difference between the genotypic value of the homozygote with greater expression and the mean of the genotypic values of the homozygotes) and d (due to dominance). Only in simple genetic systems or in those where complementary, recessive, dominant and recessive, duplicate genes with cumulative effects, or non-epistatic gene interaction types of epistasis predominate, with high frequencies of dominant genes, epistatic components can be of reduced or negligible magnitude.


Key words: genic interaction, digenic epistasis, genetic parameters, non-inbred populations.

## INTRODUCTION

Although from the biological point of view the presence of epistasis is the rule and not the exception, as has been shown in many studies on the inheritance of qualitative traits, its study in quantitative genetic systems is one of the most complex problems faced by geneticists. This is not due to lack of theoretical knowledge, that was fully established almost 50 years ago by Cockerham (1954) and Kempthorne (1955), nor to the lack of methodologies, such as generation mean analysis (Mather and Jinks 1974) and triple test cross (Kearsey and Poon 1996), but rather to the infinite possibilities of genetic systems considering the combinations of degrees of dominance, gene frequencies and number of genes. Without exception, in the top technical publications on Quantitative Genetics (Kempthorne 1973, Hallauer and Miranda Filho 1988, Wricke and Weber 1986, Comstock 1996, Lynch and Walsh 1996) and in other published studies of quantitative trait inheritance that include epistasis, there is no assessment of
relative importance of the epistatic components of genotypic variance, or of their contribution to gains from selection or of the bias in estimating genetic parameters due to fitting the simple additive-dominant model. The objective of this study was to contribute with theoretical information on the relative magnitudes of the epistatic components of genotypic variance, considering digenic epistasis.

## METHODS

All theory of polygenic systems that include interaction effects between non-allelic genes was developed by Cockerham (1954) and Kempthorne (1955), although some differences in modeling exist. In those two articles, the components of genotypic variance assuming epistasis were described. Furthermore, the authors developed expressions describing the covariance between genotypic values of relatives. Cockerham's model is the regression of the genotypic value of an individual (G) as function of the

[^0]number of genes that increase the trait expression $\left(\mathrm{X}_{1}\right.$ and $\left.\mathrm{X}_{2}\right)$, which can be defined as
\[

$$
\begin{aligned}
& G=\beta_{0}+\beta_{1} X_{1}+\beta_{2} X_{1}^{2}+\beta_{3} X_{2}+\beta_{4} X_{2}^{2}+ \\
& \beta_{5} X_{1} X_{2}+\beta_{6} X_{1} X_{2}^{2}+\beta_{7} X_{1}^{2} X_{2}+\beta_{8} X_{1}^{2} X_{2}^{2}
\end{aligned}
$$
\]

In matrix terms

$$
\left[\begin{array}{l}
G_{22} \\
G_{12} \\
G_{02} \\
G_{21} \\
\mathrm{G}_{11} \\
\mathrm{G}_{01} \\
\mathrm{G}_{20} \\
\mathrm{G}_{10} \\
\mathrm{G}_{00}
\end{array}\right]=\left[\begin{array}{lllllllll}
1 & 2 & 4 & 2 & 4 & 4 & 8 & 8 & 16 \\
1 & 1 & 1 & 2 & 4 & 2 & 4 & 2 & 4 \\
1 & 0 & 0 & 2 & 4 & 0 & 0 & 0 & 0 \\
1 & 2 & 4 & 1 & 1 & 2 & 2 & 4 & 4 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 2 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]\left[\begin{array}{l}
\beta_{0} \\
\beta_{1} \\
\beta_{2} \\
\beta_{3} \\
\beta_{4} \\
\beta_{5} \\
\beta_{6} \\
\beta_{7} \\
\beta_{8}
\end{array}\right]
$$

where $\mathrm{G}_{\mathrm{ij}}$ is the genotypic value of the carrier of i copies of gene $A$ and $j$ copies of gene $B(i, j=0,1$ or 2$)$ which increase trait expression.

Orthogonalizing the columns of $\mathrm{X}(\mathrm{Q})$ by the orthogonal polynomials technique, results in

$$
\begin{aligned}
& \mathrm{P}_{0}=\mathrm{Q}_{0} \\
& \mathrm{P}_{1}=\mathrm{Q}_{1}-\frac{\mathrm{Q}_{1}^{\prime} \mathrm{AP}_{0}}{\mathrm{P}_{0}^{\prime} \mathrm{AP}_{0}} \cdot \mathrm{P}_{0} \\
& \mathrm{P}_{2}=\mathrm{Q}_{2}-\frac{\mathrm{Q}_{2}^{\prime} \mathrm{AP}_{0}}{\mathrm{P}_{0}^{\prime} \mathrm{AP} \mathrm{P}_{0}} \cdot \mathrm{P}_{0}-\frac{\mathrm{Q}_{2}^{\prime} \mathrm{AP}_{1}}{\mathrm{P}_{1}^{\prime} \mathrm{AP}{ }_{1}} \cdot \mathrm{P}_{1} \\
& \mathrm{P}_{8}=\mathrm{Q}_{8}-\frac{\mathrm{Q}_{8}^{\prime} A \mathrm{AP}_{0}}{\mathrm{P}_{0}^{\prime} A \mathrm{AP}_{0}} \cdot \mathrm{P}_{0}-\frac{\mathrm{Q}_{8}^{\prime} \mathrm{AP} P_{1}}{\mathrm{P}_{1}^{\prime} \mathrm{AP}} \mathrm{P}_{1} \cdot \mathrm{P}_{1}-\frac{\mathrm{Q}_{8}^{\prime} \mathrm{AP} P_{2}}{\mathrm{P}_{2}^{\prime} \mathrm{AP}_{2}} \cdot \mathrm{P}_{2}-\frac{\mathrm{Q}_{8}^{\prime} \mathrm{AP}_{3}}{\mathrm{P}_{3}^{\prime} \mathrm{AP}_{3}} \cdot \mathrm{P}_{3} \\
& -\frac{\mathrm{Q}_{8}^{\prime} \mathrm{AP}_{4}}{\mathrm{P}_{4}^{\prime} \mathrm{AP}_{4}} \cdot \mathrm{P}_{4}-\frac{\mathrm{Q}_{8}^{\prime} \mathrm{AP}_{5}}{\mathrm{P}_{5}^{\prime} \mathrm{AP}_{5}} \cdot \mathrm{P}_{5}-\frac{\mathrm{Q}_{8}^{\prime} \mathrm{AP}_{6}}{\mathrm{P}_{6}^{\prime} \mathrm{AP}_{6}} \cdot \mathrm{P}_{6}-\frac{\mathrm{Q}_{8}^{\prime} \mathrm{AP}_{7}}{\mathrm{P}_{7}^{\prime} \mathrm{AP}_{7}} \cdot \mathrm{P}_{7}
\end{aligned}
$$

where A is a diagonal matrix of probabilities of the genotypes $\left(\mathrm{f}_{\mathrm{ij}}\right)$. Assuming non-inbred population in Hardy-Weinberg equilibrium and in linkage equilibrium,

$$
\begin{aligned}
\mathrm{f}_{22} & =\mathrm{p}_{\mathrm{a}}^{2} \mathrm{p}_{\mathrm{b}}^{2} \\
\mathrm{f}_{12} & =2 \mathrm{paq}_{\mathrm{a}} \mathrm{p}_{\mathrm{b}}^{2} \\
\mathrm{f}_{02} & =\mathrm{q}_{\mathrm{a}}^{2} \mathrm{p}_{\mathrm{b}}^{2} \\
\mathrm{f}_{21} & =2 \mathrm{p}_{\mathrm{a}}^{2} \mathrm{p}_{\mathrm{b}} \mathrm{q}_{\mathrm{b}} \\
\mathrm{f}_{11} & =4 \mathrm{paqa}_{\mathrm{b}} \mathrm{q}_{\mathrm{b}} \\
\mathrm{f}_{01} & =2 \mathrm{q}_{\mathrm{a}}^{2} \mathrm{p}_{\mathrm{b}} \mathrm{q}_{\mathrm{b}} \\
\mathrm{f}_{20} & =\mathrm{p}_{\mathrm{a}}^{2} \mathrm{q}_{\mathrm{b}}^{2} \\
\mathrm{f}_{10} & =2 \mathrm{p}_{\mathrm{aq}} \mathrm{q}_{\mathrm{a}}^{2} \\
\mathrm{f}_{00} & =\mathrm{q}_{\mathrm{a}}^{2} \mathrm{q}_{\mathrm{b}}^{2}
\end{aligned}
$$

where p is the frequency of the gene that increases the trait expression.

The genotypic values are
$\mathrm{G}_{22}=\left(\mathrm{m}_{\mathrm{a}}+\mathrm{a}_{\mathrm{a}}\right)+\left(\mathrm{m}_{\mathrm{b}}+\mathrm{a}_{\mathrm{b}}\right)+\mathrm{I}_{22}=\left(\mathrm{M}_{\mathrm{a}}+\mathrm{A}_{\mathrm{AA}}+\mathrm{D}_{\mathrm{AA}}\right)+\left(\mathrm{M}_{\mathrm{b}}+\mathrm{A}_{\mathrm{BB}}+\mathrm{D}_{\mathrm{BB}}\right)+\mathrm{I}_{22}$ $\mathrm{G}_{12}=\left(\mathrm{m}_{\mathrm{a}}+\mathrm{d}_{\mathrm{a}}\right)+\left(\mathrm{m}_{\mathrm{b}}+\mathrm{a}_{\mathrm{b}}\right)+\mathrm{I}_{12}=\left(\mathrm{M}_{\mathrm{a}}+\mathrm{A}_{\mathrm{Aa}}+\mathrm{D}_{\mathrm{Aa}}\right)+\left(\mathrm{M}_{\mathrm{b}}+\mathrm{A}_{\mathrm{BB}}+\mathrm{D}_{\mathrm{BB}}\right)+\mathrm{I}_{12}$ $\mathrm{G}_{02}=\left(\mathrm{m}_{\mathrm{a}}-\mathrm{a}_{\mathrm{a}}\right)+\left(\mathrm{m}_{\mathrm{b}}+\mathrm{a}_{\mathrm{b}}\right)+\mathrm{I}_{02}=\left(\mathrm{M}_{\mathrm{a}}+\mathrm{A}_{\mathrm{aa}}+\mathrm{D}_{\mathrm{aa}}\right)+\left(\mathrm{M}_{\mathrm{b}}+\mathrm{A}_{\mathrm{BB}}+\mathrm{D}_{\mathrm{BB}}\right)+\mathrm{I}_{02}$ $\mathrm{G}_{21}=\left(\mathrm{m}_{\mathrm{a}}+\mathrm{a}_{\mathrm{a}}\right)+\left(\mathrm{m}_{\mathrm{b}}+\mathrm{d}_{\mathrm{b}}\right)+\mathrm{I}_{21}=\left(\mathrm{M}_{\mathrm{a}}+\mathrm{A}_{\mathrm{AA}}+\mathrm{D}_{\mathrm{AA}}\right)+\left(\mathrm{M}_{\mathrm{b}}+\mathrm{A}_{\mathrm{Bb}}+\mathrm{D}_{\mathrm{Bb}}\right)+\mathrm{I}_{21}$ $\mathrm{G}_{11}=\left(\mathrm{m}_{\mathrm{a}}+\mathrm{d}_{\mathrm{a}}\right)+\left(\mathrm{m}_{\mathrm{b}}+\mathrm{d}_{\mathrm{b}}\right)+\mathrm{I}_{11}=\left(\mathrm{M}_{\mathrm{a}}+\mathrm{A}_{\mathrm{Aa}}+\mathrm{D}_{\mathrm{Aa}}\right)+\left(\mathrm{M}_{\mathrm{b}}+\mathrm{A}_{\mathrm{Bb}}+\mathrm{D}_{\mathrm{Bb}}\right)+\mathrm{I}_{11}$ $\mathrm{G}_{01}=\left(\mathrm{m}_{\mathrm{a}}-\mathrm{a}_{\mathrm{a}}\right)+\left(\mathrm{m}_{\mathrm{b}}+\mathrm{d}_{\mathrm{b}}\right)+\mathrm{I}_{01}=\left(\mathrm{M}_{\mathrm{a}}+\mathrm{A}_{\mathrm{aa}}+\mathrm{D}_{\mathrm{aa}}\right)+\left(\mathrm{M}_{\mathrm{b}}+\mathrm{A}_{\mathrm{Bb}}+\mathrm{D}_{\mathrm{Bb}}\right)+\mathrm{I}_{01}$ $\mathrm{G}_{20}=\left(\mathrm{m}_{\mathrm{a}}+\mathrm{a}_{\mathrm{a}}\right)+\left(\mathrm{m}_{\mathrm{b}}-\mathrm{a}_{\mathrm{b}}\right)+\mathrm{I}_{20}=\left(\mathrm{M}_{\mathrm{a}}+\mathrm{A}_{\mathrm{AA}}+\mathrm{D}_{\mathrm{AA}}\right)+\left(\mathrm{M}_{\mathrm{b}}+\mathrm{A}_{\mathrm{bb}}+\mathrm{D}_{\mathrm{bb}}\right)+\mathrm{I}_{20}$ $\mathrm{G}_{10}=\left(\mathrm{m}_{\mathrm{a}}+\mathrm{d}_{\mathrm{a}}\right)+\left(\mathrm{m}_{\mathrm{b}}-\mathrm{a}_{\mathrm{b}}\right)+\mathrm{I}_{10}=\left(\mathrm{M}_{\mathrm{a}}+\mathrm{A}_{\mathrm{Aa}}+\mathrm{D}_{\mathrm{Aa}}\right)+\left(\mathrm{M}_{\mathrm{b}}+\mathrm{A}_{\mathrm{bb}}+\mathrm{D}_{\mathrm{bb}}\right)+\mathrm{I}_{10}$ $\mathrm{G}_{00}=\left(\mathrm{m}_{\mathrm{a}}-\mathrm{a}_{\mathrm{a}}\right)+\left(\mathrm{m}_{\mathrm{b}}-\mathrm{a}_{\mathrm{b}}\right)+\mathrm{I}_{00}=\left(\mathrm{M}_{\mathrm{a}}+\mathrm{A}_{\mathrm{aa}}+\mathrm{D}_{\mathrm{aa}}\right)+\left(\mathrm{M}_{\mathrm{b}}+\mathrm{A}_{\mathrm{bb}}+\mathrm{D}_{\mathrm{bb}}\right)+\mathrm{I}_{00}$
where, for each gene, $\mathbf{m}$ is the mean of the genotypic values of the homozygotes, $\mathbf{a}$ is the deviation between the genotypic value of the homozygote with greatest expression and $\mathbf{m}, \mathbf{d}$ is the deviation due to dominance, $\mathbf{M}$ is the population mean, $\mathbf{A}$ is additive genetic value and $\mathbf{D}$ is genetic value due to dominance. Regarding one gene, $M=m+(p-q) a+2 p q d, A=2 q \alpha$ and $D=$ $-2 q^{2} d$, if the individual is homozygous for the gene that increases the trait expression, or $A=(q-p) \alpha$ and $D=2 p q d$, if the individual is heterozygous, or $A=-2 p \alpha$ and $D=-2 p^{2} d$, if the individual is homozygous for the gene that decreases the trait expression, where $\alpha$ is the average effect of a gene substitution (Falconer and Mackay 1996).

Assuming in relation to the epistatic genetic values (I),

$$
\begin{aligned}
& \overline{\mathrm{I}}_{2 .}=\mathrm{p}_{\mathrm{b}}^{2} \mathrm{I}_{22}+2 \mathrm{p}_{\mathrm{b}} \mathrm{q}_{\mathrm{b}} \mathrm{I}_{21}+\mathrm{q}_{\mathrm{b}}^{2} \mathrm{I}_{20}=\overline{\mathrm{I}}_{1 .}=\overline{\mathrm{I}}_{0}=0 \\
& \overline{\mathrm{I}}_{2}=\mathrm{pa}_{\mathrm{a}}^{2} \mathrm{I}_{22}+2 \mathrm{paq}_{\mathrm{a}} \mathrm{I}_{12}+\mathrm{q}_{\mathrm{a}}^{2} \mathrm{I}_{02}=\overline{\mathrm{I}}_{1}=\overline{\mathrm{I}}_{0}=0 \\
& \mathrm{I}_{\mathrm{U}}=\mathrm{p}_{\mathrm{a}}^{2} \mathrm{p}_{\mathrm{b}}^{2} \mathrm{I}_{22}+\cdots+\mathrm{q}_{\mathrm{a}}^{2} \mathrm{q}_{\mathrm{b}}^{2} \mathrm{I}_{00}=0
\end{aligned}
$$

the reductions in the total sum of squares are

$$
\begin{aligned}
& R\left(\beta_{0}\right)=\frac{\left(P_{0}^{\prime} A Y\right)^{2}}{P_{0}^{\prime} A P_{0}}=M^{2} \\
& R\left(\beta_{1} \mid \beta_{0}\right)=\frac{\left(P_{1}^{\prime} A Y\right)^{2}}{P_{1}^{\prime} A_{1}}=2 p_{a} q_{a} \alpha_{a}^{2}=\sigma_{A_{a}}^{2} \\
& R\left(\beta_{2} \mid \beta_{0}, \beta_{1}\right)=\frac{\left(P_{2}^{\prime} A Y\right)^{2}}{P_{2}^{\prime} A P_{2}}=4 p_{a}^{2} q_{a}^{2} d_{a}^{2}=\sigma_{D_{a}}^{2} \\
& R\left(\beta_{3} \mid \beta_{0}\right)=\frac{\left(P_{3}^{\prime} A Y\right)^{2}}{P_{3}^{\prime} A_{3}}=2 p_{b} q_{b} \alpha_{b}^{2}=\sigma_{A_{b}}^{2} \\
& R\left(\beta_{4} \mid \beta_{0}, \beta_{3}\right)=\frac{\left(P_{4}^{\prime} A Y\right)^{2}}{P_{4}^{\prime} A P_{4}}=4 p_{b}^{2} q_{b}^{2} d_{b}^{2}=\sigma_{D_{b}}^{2}
\end{aligned}
$$

The transformed X matrix is

$$
\mathrm{R}\left(\beta_{5} \mid \beta_{0}, \beta_{1}, \beta_{3}\right)=4 \mathrm{p}_{\mathrm{a}} \mathrm{q}_{\mathrm{a}} \mathrm{p}_{\mathrm{b}} \mathrm{q}_{\mathrm{b}}\left(\mathrm{p}_{\mathrm{a}} \mathrm{p}_{\mathrm{b}} \mathrm{e}_{22}+\mathrm{p}_{\mathrm{a}} \mathrm{q}_{\mathrm{b}} \mathrm{e}_{21}+\mathrm{q}_{\mathrm{a}} \mathrm{p}_{\mathrm{b}} \mathrm{e}_{12}+\mathrm{q}_{\mathrm{a}} \mathrm{q}_{\mathrm{b}} \mathrm{e}_{11}\right)^{2}=\sigma_{A A}^{2}
$$

$$
R\left(\beta_{6} \mid \beta_{0}, \beta_{1}, \beta_{3}, \beta_{4}, \beta_{5}\right)=2 p_{a} q_{a} p_{b}^{2} q_{b}^{2}\left(p_{a} e_{22}-p_{a} e_{21}+q_{a} e_{12}-q_{a} e_{11}\right)^{2}=\sigma_{A D}^{2}
$$

$$
\mathrm{R}\left(\beta_{7} \mid \beta_{0}, \beta_{1}, \beta_{2}, \beta_{3}, \beta_{5}\right)=2 \mathrm{p}_{\mathrm{a}}^{2} q_{a}^{2} \mathrm{p}_{\mathrm{b}} q_{\mathrm{b}}\left(\mathrm{p}_{\mathrm{b}} \mathrm{e}_{22}+\mathrm{q}_{\mathrm{b}} \mathrm{e}_{21}-\mathrm{p}_{\mathrm{b}} \mathrm{e}_{12}-\mathrm{q}_{\mathrm{b}} \mathrm{e}_{11}\right)^{2}=\sigma_{D A}^{2}
$$

$$
\mathrm{R}\left(\beta_{8} \mid \beta_{0}, \beta_{1}, \beta_{2}, \beta_{3}, \beta_{4}, \beta_{5}, \beta_{6}, \beta_{7}\right)=\mathrm{p}_{\mathrm{a}}^{2} \mathrm{q}_{\mathrm{a}}^{2} \mathrm{p}_{\mathrm{b}}^{2} \mathrm{q}_{\mathrm{b}}^{2}\left(\mathrm{e}_{22}-\mathrm{e}_{21}-\mathrm{e}_{12}+\mathrm{e}_{11}\right)^{2}=\sigma_{\mathrm{DD}}^{2}
$$

where

$$
\begin{aligned}
& \mathrm{e}_{22}=\mathrm{G}_{22}-\mathrm{G}_{21}-\mathrm{G}_{12}+\mathrm{G}_{11}=\mathrm{I}_{22}-\mathrm{I}_{21}-\mathrm{I}_{12}+\mathrm{I}_{11} \\
& \mathrm{e}_{21}=\mathrm{G}_{21}-\mathrm{G}_{20}-\mathrm{G}_{11}+\mathrm{G}_{10}=\mathrm{I}_{21}-\mathrm{I}_{20}-\mathrm{I}_{11}+\mathrm{I}_{10} \\
& \mathrm{e}_{12}=\mathrm{G}_{12}-\mathrm{G}_{11}-\mathrm{G}_{02}+\mathrm{G}_{01}=\mathrm{I}_{12}-\mathrm{I}_{11}-\mathrm{I}_{02}+\mathrm{I}_{01} \\
& \mathrm{e}_{11}=\mathrm{G}_{11}-\mathrm{G}_{10}-\mathrm{G}_{01}+\mathrm{G}_{00}=\mathrm{I}_{11}-\mathrm{I}_{10}-\mathrm{I}_{01}+\mathrm{I}_{00}
\end{aligned}
$$

The genotypic variance is

$$
\sigma_{\mathrm{G}}^{2}=\mathrm{Y}^{\prime} \mathrm{AY}-\mathrm{R}\left(\beta_{0}\right)=\sigma_{\mathrm{A}}^{2}+\sigma_{\mathrm{D}}^{2}+\sigma_{\mathrm{AA}}^{2}+\sigma_{\mathrm{AD}}^{2}+\sigma_{\mathrm{DA}}^{2}+\sigma_{\mathrm{DD}}^{2}
$$

Considering k genes,

$$
\begin{aligned}
& \sigma_{A A}^{2}=4 \sum_{i=1<j=1}^{k} \sum_{j=1}^{k} p_{i} q_{i} p_{j} q_{j}\left(p_{i p} p_{j} 22_{i j}+p_{i q} q_{j} e_{21_{i j}}+q_{i} p_{j e} 1_{i j}+q_{i q} q_{j} e_{11_{i j}}\right)= \\
& \sigma_{A D}^{2}=2 \sum_{i=1<j}^{k} \sum_{j=1}^{k} p_{i} q_{i} p_{j}^{2} q_{j}^{2}\left(p_{i} e_{22_{i j}}-p_{i} e_{21} 1_{i j}+q_{i} e_{12_{i j}}-q_{i} e_{11_{i j}}\right)^{2} \\
& \sigma_{D A}^{2}=2 \sum_{i=1<j}^{k} \sum_{j=1}^{k} p_{i}^{2} q_{i}^{2} p_{j q} q_{j}\left(p_{j e} 22_{i j}+q_{j} e_{21} 1_{i j}-p_{j} e_{12_{i j}}-q_{j e} 1_{1 i j}\right)^{2} \\
& \sigma_{D D}^{2}=\sum_{i=1<j=1}^{k} \sum_{i}^{k} p_{i}^{2} q_{i}^{2} p_{j}^{2} q_{j}^{2}\left(e_{22_{i j}}-e_{21_{i j}}-e_{12}{ }_{i j}+e_{11_{i j}}\right)^{2}
\end{aligned}
$$

where $\sigma_{\mathrm{AA}}^{2}, \sigma_{\mathrm{AD}}^{2}, \sigma_{\mathrm{DA}}^{2}$ and $\sigma_{\mathrm{DD}}^{2}$ are the additive x additive, additive x dominant, dominant x additive and dominant x dominant epistatic genetic variances, respectively.

## Types of Epistasis

The complexity of the polygenic systems makes any theoretical study of epistasis very difficult because there are infinite combinations of gene frequencies and degrees of dominance. To simplify, known types of digenic epistasis will be considered, characterized as follows:

- Complementary epistasis (9:7): $\mathrm{a}_{\mathrm{i}}=\mathrm{d}_{\mathrm{i}}=\mathrm{a}$ and $\mathrm{p}_{\mathrm{i}}=\mathrm{p}$, for every i, and

$$
\begin{aligned}
& \mathrm{I}_{00}=2\left(1-\mathrm{q}^{2}\right) \mathrm{a} \\
& \mathrm{I}_{20}=\mathrm{I}_{10}=\mathrm{I}_{02}=\mathrm{I}_{01}=-2 \mathrm{q}^{2} \mathrm{a} \\
& \mathrm{I}_{22}=\mathrm{I}_{21}=\mathrm{I}_{12}=\mathrm{I}_{11}=\frac{2 \mathrm{q}^{4} \mathrm{a}}{1-\mathrm{q}^{2}}
\end{aligned}
$$

- Duplicate epistasis (15:1): $a_{i}=d_{i}=a$ and $p_{i}=p$, for every $i$, and

$$
\begin{aligned}
& I_{00}=-\frac{2\left(1-q^{2}\right)^{2} a}{q^{2}} \\
& I_{20}=I_{10}=I_{02}=I_{01}=2\left(1-q^{2}\right) a \\
& I_{22}=I_{21}=I_{12}=I_{11}=-2 q^{2} a
\end{aligned}
$$

- Dominant and recessive epistasis (13:3): $a_{a}=d_{a}=-a_{b}=-d_{b}, p_{a}=$ $\mathrm{p}_{\mathrm{b}}=\mathrm{p}$ and

$$
\begin{aligned}
& \mathrm{I}_{00}=\mathrm{I}_{22}=\mathrm{I}_{21}=\mathrm{I}_{12}=\mathrm{I}_{11}=\mathrm{a}_{\mathrm{a}} \\
& \mathrm{I}_{20}=\mathrm{I}_{10}=-\mathrm{a}_{\mathrm{a}} \\
& \mathrm{I}_{02}=\mathrm{I}_{01}=\theta \mathrm{a}_{\mathrm{a}}
\end{aligned}
$$

where $\theta$ is a coefficient of proportionality.

- Recessive epistasis (9:3:4): $\mathrm{a}_{\mathrm{a}}=\mathrm{d}_{\mathrm{a}}=\theta^{\prime} \mathrm{a}_{\mathrm{b}}=\theta^{\prime} \mathrm{d}_{\mathrm{b}}$, where $\theta^{\prime}$ is a coefficient of proportionality, different from $1, p_{a}=p_{b}=p$ and

$$
\begin{aligned}
& \mathrm{I}_{00}=2\left(1-\mathrm{q}^{2}\right) \mathrm{a}_{\mathrm{b}} \\
& \mathrm{I}_{20}=\mathrm{I}_{10}=\mathrm{I}_{02}=\mathrm{I}_{01}=-2 \mathrm{q}^{2} \mathrm{a}_{\mathrm{b}} \\
& \mathrm{I}_{22}=\mathrm{I}_{21}=\mathrm{I}_{12}=\mathrm{I}_{11}=\frac{2 \mathrm{q}^{4} \mathrm{a}_{\mathrm{b}}}{1-\mathrm{q}^{2}}
\end{aligned}
$$

$\cdot$ Dominant epistasis (12:3:1): $\mathrm{a}_{\mathrm{a}}=\mathrm{d}_{\mathrm{a}}=\theta^{\prime} \mathrm{a}_{\mathrm{b}}=\theta^{\prime} \mathrm{d}_{\mathrm{b}}, \mathrm{p}_{\mathrm{a}}=\mathrm{p}_{\mathrm{b}}=$ p and

$$
\begin{aligned}
& I_{00}=-\frac{2\left(1-q^{2}\right)^{2} a_{b}}{q^{2}} \\
& I_{20}=I_{10}=I_{02}=I_{01}=2\left(1-q^{2}\right) a_{b} \\
& I_{22}=I_{21}=I_{12}=I_{11}=-2 q^{2} a_{b}
\end{aligned}
$$

- Duplicate genes with cumulative effects (9:6:1): $a_{i}=d_{i}$ e $p_{i}$ $=p$, for every $i$, and

$$
\begin{aligned}
& I_{00}=\theta a \\
& I_{20}=I_{10}=I_{02}=I_{01}=-\frac{q^{2} \theta a}{1-q^{2}} \\
& I_{22}=I_{21}=I_{12}=I_{11}=\frac{q^{4} \theta a}{\left(1-q^{2}\right)^{2}}
\end{aligned}
$$

where $\theta$ is a proportionality coefficient.

- Non epistatic genic interaction (9:3:3:1): $a_{a}=d_{a}=\theta^{\prime} a_{b}=$ $\theta^{\prime} \mathrm{d}_{\mathrm{b}}, \mathrm{p}_{\mathrm{a}}=\mathrm{p}_{\mathrm{b}}=\mathrm{p}$ and

$$
\begin{aligned}
& I_{00}=\theta a_{a} \\
& I_{20}=I_{10}=I_{02}=I_{01}=-\frac{q^{2} \theta a_{a}}{1-q^{2}} \\
& I_{22}=I_{21}=I_{12}=I_{11}=\frac{q^{4} \theta a_{a}}{\left(1-q^{2}\right)^{2}}
\end{aligned}
$$

## RESULTS AND DISCUSSION

Assuming complementary epistasis and population with reduced frequencies of dominant genes (Table 1), the additive x additive epistatic genetic variance is, in general, the most important component of the genotypic variance independently of the polygenic system, surpassing the additive and dominance genetic variances. Only in simpler systems, with few interacting genes, the additive variance can be the component of largest magnitude. In spite of complete dominance, the variances due to dominance and additive x dominant, dominant x additive, and dominant x dominant epistatic effects represent at most $10 \%$ of the genotypic variance. In populations with intermediate allelic frequencies (Table 1) and in polygenic systems with up to 100 genes, if about $20 \%$ of these genes interact, the additive variance is the component of largest magnitude of the genotypic variance. In more complex systems, it is surpassed by the additive x additive variance. Comparatively to the populations with low frequencies of dominant genes, there is, in general, a decrease in the magnitude of the additive x additive epistatic variance and an increase in the magnitude of the other components of the genotypic variance. Even so, it continues to be the most
important component of the epistatic variance. Magnitudes of the epistatic variances are lower. If the frequencies of dominant genes in the population are high (Table 1), with the exception of the very complex polygenic systems with at least 1000 genes, of which at least $50 \%$ interact, the variance due to dominance is the component of largest magnitude of the genotypic variance. The most relevant component of the epistatic variance is the dominant x dominant variance. With complementary epistasis, the magnitude of the epistatic variance is maximized when the frequencies of the dominant genes are reduced and minimized when the frequencies of the dominant genes are high. When the gene frequencies are intermediary, the additive $x$ additive variance has the same magnitude as the sum of the additive x dominant and dominant x additive epistatic variances. Regardless of the gene frequencies, the increase in the number of genes in the polygenic system and in the number of interacting genes will increase the magnitude of the epistatic variance, which can represent practically $100 \%$ of the genotypic variance.

Admitting duplicate epistasis (Table 2), if the frequencies of the dominant genes in the population are small, only in simpler polygenic systems of at most 100 genes, with about $20 \%$ of them interacting, the magnitude of the additive variance

Table 1. Relative magnitudes (\%) of the components of genotypic variance, considering complementary epistasis, in relation to different polygenic systems and populations

| k | $k^{\prime}$ | p | $\sigma_{A}^{2} / \sigma_{G}^{2}$ | $\sigma_{D}^{2} / \sigma_{G}^{2}$ | $\sigma_{\mathrm{AA}}^{2} / \sigma_{\mathrm{G}}^{2}$ | $\left(\sigma_{\mathbf{A D}}^{2}+\sigma_{\mathbf{D A}}^{2}\right) / \sigma_{G}^{2}$ | $\sigma_{\mathrm{DD}}^{2} / \sigma_{\mathrm{G}}^{2}$ | $\sigma_{\mathrm{I}}^{2} / \sigma_{\mathrm{G}}^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 2 | 0.1 | 30.252 | 1.681 | 61.091 | 6.788 | 0.189 | 68.067 |
| 10 | 2 | 0.1 | 66.421 | 3.690 | 26.826 | 2.981 | 0.083 | 29.889 |
| 10 | 5 | 0.1 | 18.000 | 1.000 | 72.698 | 8.078 | 0.224 | 81.000 |
| 10 | 10 | 0.1 | 4.694 | 0.261 | 85.304 | 9.478 | 0.263 | 95.046 |
| 100 | 20 | 0.1 | 10.411 | 0.578 | 79.888 | 8.876 | 0.247 | 89.011 |
| 100 | 50 | 0.1 | 1.780 | 0.099 | 88.064 | 9.785 | 0.272 | 98.121 |
| 100 | 100 | 0.1 | 0.447 | 0.025 | 89.327 | 9.925 | 0.276 | 99.528 |
| 1000 | 200 | 0.1 | 1.104 | 0.061 | 88.705 | 9.856 | 0.274 | 98.835 |
| 1000 | 500 | 0.1 | 0.178 | 0.010 | 89.582 | 9.954 | 0.276 | 99.812 |
| 1000 | 1000 | 0.1 | 0.044 | 0.002 | 89.709 | 9.968 | 0.277 | 99.953 |
| 2 | 2 | 0.5 | 57.143 | 28.571 | 6.349 | 6.349 | 1.587 | 14.286 |
| 10 | 2 | 0.5 | 64.516 | 32.258 | 1.434 | 1.434 | 0.358 | 3.226 |
| 10 | 5 | 0.5 | 50.000 | 25.000 | 11.111 | 11.111 | 2.778 | 25.000 |
| 10 | 10 | 0.5 | 26.667 | 13.333 | 26.667 | 26.667 | 6.667 | 60.000 |
| 100 | 20 | 0.5 | 40.816 | 20.408 | 17.234 | 17.234 | 4.308 | 38.776 |
| 100 | 50 | 0.5 | 13.115 | 6.557 | 35.701 | 35.701 | 8.925 | 80.328 |
| 100 | 100 | 0.5 | 3.810 | 1.905 | 41.905 | 41.905 | 10.476 | 94.286 |
| 1000 | 200 | 0.5 | 8.734 | 4.367 | 38.622 | 38.622 | 9.656 | 86.900 |
| 1000 | 500 | 0.5 | 1.566 | 0.783 | 43.401 | 43.401 | 10.850 | 97.652 |
| 1000 | 1000 | 0.5 | 0.398 | 0.199 | 44.179 | 44.179 | 11.045 | 99.403 |
| 2 | 2 | 0.9 | 18.090 | 81.407 | 0.017 | 0.150 | 0.336 | 0.503 |
| 10 | 2 | 0.9 | 18.163 | 81.736 | 0.003 | 0.030 | 0.068 | 0.101 |
| 10 | 5 | 0.9 | 18.000 | 81.000 | 0.033 | 0.298 | 0.669 | 1.000 |
| 10 | 10 | 0.9 | 17.391 | 78.261 | 0.144 | 1.294 | 2.911 | 4.348 |
| 100 | 20 | 0.9 | 17.839 | 80.278 | 0.062 | 0.560 | 1.261 | 1.883 |
| 100 | 50 | 0.9 | 16.180 | 72.809 | 0.364 | 3.276 | 7.371 | 11.011 |
| 100 | 100 | 0.9 | 12.121 | 54.545 | 1.102 | 9.917 | 22.314 | 33.333 |
| 1000 | 200 | 0.9 | 15.139 | 68.124 | 0.553 | 4.980 | 11.204 | 16.737 |
| 1000 | 500 | 0.9 | 8.045 | 36.201 | 1.843 | 16.588 | 37.323 | 55.754 |
| 1000 | 1000 | 0.9 | 3.008 | 13.534 | 2.759 | 24.831 | 55.869 | 83.459 |

[^1]will be equal or larger to the additive x additive variance. In more complex systems, the additive x additive variance is the largest component of the genotypic variance. Regarding populations with intermediary allelic frequencies, the additive x additive variance is the component of largest magnitude. Exceptions are simple systems of up to 10 genes, with about $20 \%$ of them interacting, where the additive x additive variance will be smaller than the additive variance (Table 2). In populations with high frequencies of dominant genes, the variance of largest magnitude is the dominant x dominant regardless of the polygenic system, representing 61 to $67 \%$ of the genotypic variance (Table 2). Contrary to that observed with complementary epistasis, the variance due to dominance is of negligible magnitude. With duplicate epistasis, an increase in the frequencies of dominant genes will increase the magnitude of the epistatic variance, which can represent almost $100 \%$ of the genotypic variance. In populations with intermediary allelic frequencies, the additive x additive variance is also equal to the sum of the additive x dominant and dominant x additive variances. As described for complementary epistasis, an increase in the trait control complexity and in the number of interacting genes can bring the proportion of the genotypic variance closer to $100 \%$ due
to the differences among the epistatic genetic values of the individuals, regardless of the gene frequencies.

Compared to the previous cases, the analysis of duplicate genes with cumulative effects is more troublesome, because for each pair of interacting genes there is a proportionality between the a deviation, constant for every gene, and the epistatic genetic value of the homozygote for the genes that reduce the trait expression ( $\theta$ ). Assuming $\theta=0.1$ and a population with reduced frequency of dominant genes, the additive x additive variance is the component of genotypic variance of largest magnitude in complex polygenic systems of at least 100 genes, with about $50 \%$ of them interacting. In simpler systems, the additive variance has a larger value (Table 3). When the allelic frequencies are intermediary, the additive variance is the component of largest magnitude followed by genetic variance due to dominance, which both correspond to practically $100 \%$ of the genotypic variance (Table 4), regardless of the polygenic system. In populations with high frequencies of dominant genes, the additive and dominance variances also represent practically $100 \%$ of the genetic variance, but the second component is the one of largest magnitude (at least $80 \%$ ), independently of the polygenic

Table 2. Relative magnitudes (\%) of the components of genotypic variance, considering duplicate epistasis, in relation to different polygenic systems and populations

| k | $\mathbf{k}^{\prime}$ | p | $\sigma_{A}^{2} / \sigma_{G}^{2}$ | $\sigma_{D}^{2} / \sigma_{G}^{2}$ | $\sigma_{\mathbf{A A}}^{2} / \sigma_{\mathbf{G}}^{2}$ | $\left(\sigma_{\mathbf{A D}}^{2}+\sigma_{\mathbf{D A}}^{2}\right) / \sigma_{\mathbf{G}}^{2}$ | $\sigma_{\mathrm{DD}}^{2} / \sigma_{\mathrm{G}}^{2}$ | $\sigma_{\mathrm{I}}^{2} / \sigma_{\mathrm{G}}^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 2 | 0.1 | 84.792 | 4.711 | 9.421 | 1.047 | 0.029 | 10.497 |
| 10 | 2 | 0.1 | 92.566 | 5.143 | 2.057 | 0.229 | 0.006 | 2.292 |
| 10 | 5 | 0.1 | 76.737 | 4.263 | 17.053 | 1.895 | 0.053 | 19.000 |
| 10 | 10 | 0.1 | 46.088 | 2.560 | 46.088 | 5.121 | 0.142 | 51.351 |
| 100 | 20 | 0.1 | 65.531 | 3.641 | 27.669 | 3.074 | 0.085 | 30.828 |
| 100 | 50 | 0.1 | 24.458 | 1.359 | 66.580 | 7.398 | 0.205 | 74.183 |
| 100 | 100 | 0.1 | 7.512 | 0.417 | 82.634 | 9.182 | 0.255 | 92.070 |
| 1000 | 200 | 0.1 | 16.715 | 0.929 | 73.916 | 8.213 | 0.228 | 82.357 |
| 1000 | 500 | 0.1 | 3.131 | 0.174 | 86.785 | 9.643 | 0.268 | 96.696 |
| 1000 | 1000 | 0.1 | 0.802 | 0.045 | 88.991 | 9.888 | 0.275 | 99.154 |
| 2 | 2 | 0.5 | 26.667 | 13.333 | 26.667 | 26.667 | 6.667 | 60.000 |
| 10 | 2 | 0.5 | 51.282 | 25.641 | 10.256 | 10.256 | 2.564 | 23.077 |
| 10 | 5 | 0.5 | 16.667 | 8.333 | 33.333 | 33.333 | 8.333 | 75.000 |
| 10 | 10 | 0.5 | 4.598 | 2.299 | 41.379 | 41.379 | 10.345 | 93.103 |
| 100 | 20 | 0.5 | 9.950 | 4.975 | 37.811 | 37.811 | 9.453 | 85.075 |
| 100 | 50 | 0.5 | 1.766 | 0.883 | 43.267 | 43.267 | 10.817 | 97.351 |
| 100 | 100 | 0.5 | 0.446 | 0.223 | 44.147 | 44.147 | 11.037 | 99.331 |
| 1000 | 200 | 0.5 | 1.098 | 0.549 | 43.712 | 43.712 | 10.928 | 98.353 |
| 1000 | 500 | 0.5 | 0.178 | 0.089 | 44.326 | 44.326 | 11.082 | 99.734 |
| 1000 | 1000 | 0.5 | 0.044 | 0.022 | 44.415 | 44.415 | 11.104 | 99.933 |
| 2 | 2 | 0.9 | 0.360 | 1.620 | 3.240 | 29.163 | 65.617 | 98.020 |
| 10 | 2 | 0.9 | 1.668 | 7.506 | 3.003 | 27.023 | 60.801 | 90.826 |
| 10 | 5 | 0.9 | 0.182 | 0.818 | 3.273 | 29.455 | 66.273 | 99.000 |
| 10 | 10 | 0.9 | 0.041 | 0.183 | 3.298 | 29.685 | 66.792 | 99.776 |
| 100 | 20 | 0.9 | 0.096 | 0.433 | 3.288 | 29.595 | 66.588 | 99.471 |
| 100 | 50 | 0.9 | 0.015 | 0.067 | 3.303 | 29.728 | 66.887 | 99.918 |
| 100 | 100 | 0.9 | 0.004 | 0.017 | 3.305 | 29.746 | 66.928 | 99.980 |
| 1000 | 200 | 0.9 | 0.009 | 0.042 | 3.304 | 29.737 | 66.908 | 99.949 |
| 1000 | 500 | 0.9 | 0.001 | 0.007 | 3.306 | 29.750 | 66.937 | 99.992 |
| 1000 | 1000 | 0.9 | 0.000 | 0.002 | 3.306 | 29.751 | 66.941 | 99.998 |

[^2]Table 3. Relative magnitudes (\%) of the components of genotypic variance, considering duplicate genes with cumulative effects and populations with reduced frequency of dominant genes (p), in different polygenic systems

| $\theta$ | k | $\mathrm{k}^{\prime}$ | p | $\sigma_{\mathbf{A}}^{2} / \sigma_{\mathbf{G}}^{2}$ | $\sigma_{\mathbf{D}}^{2} / \sigma_{\mathbf{G}}^{2}$ | $\sigma_{\mathrm{AA}}^{2} / \sigma_{\mathrm{G}}^{2}$ | $\left(\sigma_{\mathbf{A D}}^{2}+\sigma_{\mathbf{D A}}^{2}\right) / \sigma_{\mathbf{G}}^{2}$ | $\sigma_{\mathrm{DD}}^{2} / \sigma_{\mathrm{G}}^{2}$ | $\sigma_{I}^{2} / \sigma_{G}^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1 | 2 | 2 | 0.1 | 82.551 | 4.586 | 11.545 | 1.283 | 0.036 | 12.863 |
| 0.1 | 10 | 2 | 0.1 | 92.020 | 5.112 | 2.574 | 0.286 | 0.008 | 2.868 |
| 0.1 | 10 | 5 | 0.1 | 73.143 | 4.063 | 20.458 | 2.273 | 0.063 | 22.794 |
| 0.1 | 10 | 10 | 0.1 | 40.685 | 2.260 | 51.207 | 5.690 | 0.158 | 57.055 |
| 0.1 | 100 | 20 | 0.1 | 60.692 | 3.372 | 32.253 | 3.584 | 0.100 | 35.936 |
| 0.1 | 100 | 50 | 0.1 | 20.521 | 1.140 | 70.310 | 7.812 | 0.217 | 78.339 |
| 0.1 | 100 | 100 | 0.1 | 6.067 | 0.337 | 84.003 | 9.334 | 0.259 | 93.595 |
| 0.1 | 1000 | 200 | 0.1 | 13.780 | 0.766 | 76.696 | 8.522 | 0.237 | 85.455 |
| 0.1 | 1000 | 500 | 0.1 | 2.504 | 0.139 | 87.378 | 9.709 | 0.270 | 97.357 |
| 0.1 | 1000 | 1000 | 0.1 | 0.638 | 0.035 | 89.146 | 9.905 | 0.275 | 99.326 |
| 1 | 2 | 2 | 0.1 | 6.011 | 0.334 | 84.056 | 9.340 | 0.259 | 93.655 |
| 1 | 10 | 2 | 0.1 | 23.970 | 1.332 | 67.042 | 7.449 | 0.207 | 74.698 |
| 1 | 10 | 5 | 0.1 | 3.104 | 0.172 | 86.810 | 9.646 | 0.268 | 96.724 |
| 1 | 10 | 10 | 0.1 | 0.708 | 0.039 | 89.080 | 9.898 | 0.275 | 99.253 |
| 1 | 100 | 20 | 0.1 | 1.659 | 0.092 | 88.179 | 9.798 | 0.272 | 98.249 |
| 1 | 100 | 50 | 0.1 | 0.261 | 0.015 | 89.503 | 9.945 | 0.276 | 99.724 |
| 1 | 100 | 100 | 0.1 | 0.065 | 0.004 | 89.689 | 9.965 | 0.277 | 99.932 |
| 1 | 1000 | 200 | 0.1 | 0.161 | 0.009 | 89.598 | 9.955 | 0.277 | 99.830 |
| 1 | 1000 | 500 | 0.1 | 0.026 | 0.001 | 89.726 | 9.970 | 0.277 | 99.973 |
| 1 | 1000 | 1000 | 0.1 | 0.006 | 0.000 | 89.745 | 9.972 | 0.277 | 99.993 |
| 2 | 2 | 2 | 0.1 | 1.578 | 0.088 | 88.256 | 9.806 | 0.272 | 98.335 |
| 2 | 10 | 2 | 0.1 | 7.396 | 0.411 | 82.744 | 9.194 | 0.255 | 92.193 |
| 2 | 10 | 5 | 0.1 | 0.795 | 0.044 | 88.997 | 9.889 | 0.275 | 99.160 |
| 2 | 10 | 10 | 0.1 | 0.178 | 0.010 | 89.582 | 9.954 | 0.276 | 99.812 |
| 2 | 100 | 20 | 0.1 | 0.420 | 0.023 | 89.352 | 9.928 | 0.276 | 99.556 |
| 2 | 100 | 50 | 0.1 | 0.065 | 0.004 | 89.689 | 9.965 | 0.277 | 99.931 |
| 2 | 100 | 100 | 0.1 | 0.016 | 0.001 | 89.735 | 9.971 | 0.277 | 99.983 |
| 2 | 1000 | 200 | 0.1 | 0.040 | 0.002 | 89.713 | 9.968 | 0.277 | 99.957 |
| 2 | 1000 | 500 | 0.1 | 0.006 | 0.000 | 89.745 | 9.972 | 0.277 | 99.993 |
| 2 | 1000 | 1000 | 0.1 | 0.002 | 0.000 | 89.749 | 9.972 | 0.277 | 99.998 |

$\sigma_{\mathrm{I}}^{2}$ is the epistatic genetic variance; k is the number of genes and $\mathrm{k}^{\prime}$ is the number of genes that interact.
system (Table 5). If the value of $\theta$ is at least 1.0 , in the populations with reduced frequencies of dominant genes the component of largest magnitude of the genotypic variance is the additive x additive variance, regardless of the polygenic system (Table 3). If the allelic frequencies are intermediary, the additive variance is the component of greatest value in the simpler system with up to 100 genes, with about $20 \%$ of them interacting. However, the additive x additive variance is the largest component in the other cases (Table 4).

In the populations with high frequencies of dominant genes, the dominance variance is the most important component of the genotypic variance, followed by the additive variance (Table 5). In the case of duplicate genes with cumulative effects, an increase in the proportion between the a deviation and the epistatic value of the homozygote for the genes that reduced the trait expression coupled with low frequencies of the dominant genes, a high number of genes determining the trait, and a high number of interacting genes will increase the
magnitude of the epistatic variance, which can attain approximately $100 \%$ of the genotypic variance. Also in this case, when the allelic frequencies are intermediate, the additive x additive variance has the same magnitude as the sum of the additive x dominant and dominant x additive variances.

The case of dominant and recessive epistasis is even more complex. Considering three genes and digenic epistasis, only in relation to two of the three pairs it is possible to define this type of epistasis; for the third pair the epistasis is complementary, duplicate or duplicate genes with cumulative effects. The values presented in Table 6 were obtained assuming dominant and recessive epistasis of a single gene with all the $\left(k^{\prime}-1\right)$ genes remaining, theta $(\theta)$ equal to -1 and complementary epistasis in relation to the $\mathrm{k}^{\prime}\left(\mathrm{k}^{\prime}-1\right) / 2$ other pairs. In populations with reduced frequencies of dominant genes, the additive x additive variance is the component of largest magnitude of the genotypic variance, except in the simpler polygenic systems with no more than

Table 4. Relative magnitudes (\%) of the components of genotypic variance, considering duplicate genes with cumulative effects and populations with intermediary allelic frequencies (p), in different polygenic systems

| $\theta$ | k | $\mathrm{k}^{\prime}$ | p | $\sigma_{\mathbf{A}}^{2} / \sigma_{\mathbf{G}}^{2}$ | $\sigma_{\text {D }}^{\mathbf{2}} / \sigma_{\mathrm{G}}^{\mathbf{2}}$ | $\sigma_{\mathrm{AA}}^{2} / \sigma_{\mathrm{G}}^{2}$ | $\left(\sigma_{\mathrm{AD}}^{2}+\sigma_{\mathrm{DA}}^{2}\right) / \sigma_{\mathrm{G}}^{2}$ | $\sigma_{\mathrm{DD}}^{2} / \sigma_{\mathrm{G}}^{2}$ | $\sigma_{\mathrm{I}}^{2} / \sigma_{\mathrm{G}}^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1 | 2 | 2 | 0.5 | 66.617 | 33.309 | 0.033 | 0.033 | 0.008 | 0.074 |
| 0.1 | 10 | 2 | 0.5 | 66.657 | 33.328 | 0.007 | 0.007 | 0.002 | 0.015 |
| 0.1 | 10 | 5 | 0.5 | 66.568 | 33.284 | 0.066 | 0.066 | 0.016 | 0.148 |
| 0.1 | 10 | 10 | 0.5 | 66.225 | 33.113 | 0.294 | 0.294 | 0.074 | 0.662 |
| 0.1 | 100 | 20 | 0.5 | 66.480 | 33.240 | 0.125 | 0.125 | 0.031 | 0.281 |
| 0.1 | 100 | 50 | 0.5 | 65.478 | 32.739 | 0.792 | 0.792 | 0.198 | 1.782 |
| 0.1 | 100 | 100 | 0.5 | 62.112 | 31.056 | 3.037 | 3.037 | 0.759 | 6.832 |
| 0.1 | 1000 | 200 | 0.5 | 64.758 | 32.379 | 1.273 | 1.273 | 0.318 | 2.864 |
| 0.1 | 1000 | 500 | 0.5 | 56.268 | 28.134 | 6.933 | 6.933 | 1.733 | 15.599 |
| 0.1 | 1000 | 1000 | 0.5 | 38.314 | 19.157 | 18.902 | 18.902 | 4.725 | 42.529 |
| 1 | 2 | 2 | 0.5 | 62.069 | 31.034 | 3.065 | 3.065 | 0.766 | 6.897 |
| 1 | 10 | 2 | 0.5 | 65.693 | 32.847 | 0.649 | 0.649 | 0.162 | 1.460 |
| 1 | 10 | 5 | 0.5 | 58.065 | 29.032 | 5.735 | 5.735 | 1.434 | 12.903 |
| 1 | 10 | 10 | 0.5 | 40.000 | 20.000 | 17.778 | 17.778 | 4.444 | 40.000 |
| 1 | 100 | 20 | 0.5 | 52.023 | 26.012 | 9.762 | 9.762 | 2.441 | 21.965 |
| 1 | 100 | 50 | 0.5 | 23.684 | 11.842 | 28.655 | 28.655 | 7.164 | 64.474 |
| 1 | 100 | 100 | 0.5 | 8.000 | 4.000 | 39.111 | 39.111 | 9.778 | 88.000 |
| 1 | 1000 | 200 | 0.5 | 16.886 | 8.443 | 33.187 | 33.187 | 8.297 | 74.672 |
| 1 | 1000 | 500 | 0.5 | 3.422 | 1.711 | 42.163 | 42.163 | 10.541 | 94.867 |
| 1 | 1000 | 1000 | 0.5 | 0.889 | 0.444 | 43.852 | 43.852 | 10.963 | 98.667 |
| 2 | 2 | 2 | 0.5 | 51.429 | 25.714 | 10.159 | 10.159 | 2.540 | 22.857 |
| 2 | 10 | 2 | 0.5 | 62.937 | 31.469 | 2.486 | 2.486 | 0.622 | 5.594 |
| 2 | 10 | 5 | 0.5 | 41.860 | 20.930 | 16.537 | 16.537 | 4.134 | 37.209 |
| 2 | 10 | 10 | 0.5 | 18.182 | 9.091 | 32.323 | 32.323 | 8.081 | 72.727 |
| 2 | 100 | 20 | 0.5 | 31.359 | 15.679 | 23.539 | 23.539 | 5.885 | 52.962 |
| 2 | 100 | 50 | 0.5 | 8.072 | 4.036 | 39.063 | 39.063 | 9.766 | 87.892 |
| 2 | 100 | 100 | 0.5 | 2.198 | 1.099 | 42.979 | 42.979 | 10.745 | 96.703 |
| 2 | 1000 | 200 | 0.5 | 5.211 | 2.606 | 40.970 | 40.970 | 10.243 | 92.183 |
| 2 | 1000 | 500 | 0.5 | 0.890 | 0.445 | 43.851 | 43.851 | 10.963 | 98.665 |
| 2 | 1000 | 1000 | 0.5 | 0.224 | 0.112 | 44.295 | 44.295 | 11.074 | 99.663 |

$\overline{\sigma_{I}^{2}}$ is the epistatic genetic variance; $k$ is the number of genes and $k^{\prime}$ is the number of genes that interact.

10 genes, with about $20 \%$ of them interacting (Table 6). Even with complete dominance in all the loci, the genetic variances due to dominance, additive x dominant, dominant x additive and dominant x dominant epistatic effects represent together at most $10 \%$ of the genetic variance. In populations with intermediary allelic frequencies, the additive x additive variance is the component with the largest value in polygenic systems with at least 100 genes, with about $50 \%$ of them interacting (Table 6). In the other cases, it is the additive variance that represents the largest fraction of the genotypic variance, followed by dominance variance.

In populations with high frequencies of dominant genes, the component of largest magnitude is the variance due to dominance, which is surpassed by the dominant x dominant epistatic variance in the polygenic system with many genes, at least 1,000 , with at least $50 \%$ of them interacting (Table 6). As in the case of complementary epistasis, the epistatic variance is maximized when the frequencies of the dominant genes are reduced. Regardless of the gene frequencies, an increase in polygenic system complexity and in the number of interacting genes will increase the magnitude of the epistatic variance that may attain almost $100 \%$ of the genotypic variance.

Once again there is equality in the values of the additive x additive variance and the sum of the additive x dominant and dominant x additive variances when the allelic frequencies are equal.

Assuming $\mathrm{a}_{\mathrm{i}} \approx \mathrm{a}_{\mathrm{j}}(\mathrm{i}, \mathrm{j}=1, \ldots, \mathrm{k} ; \mathrm{i} \neq \mathrm{j})$, the relative values of the components of genotypic variance with recessive epistasis came close to those presented for complementary epistasis. Under the same assumption, the relative values of the components of genotypic variance with dominant epistasis came close to those presented for duplicate epistasis and the relative values of the components of the genotypic variance with non-epistasis genetic interaction approach those presented for duplicate genes with cumulative effects.

If the gene frequencies in the population are not all approximately equal, the relative magnitudes of the components of the genotypic variance differ from the presented values (Table 7). In relation to two genes, with complementary epistasis ( $\mathrm{A}_{-} \mathrm{B}$ _ with one genotypic value and $\mathrm{A}_{-} \mathrm{bb}, \mathrm{aaB}$ _ and aabb with different value), there is a tendency for the additive variance to be the component of largest value, followed by the dominance variance. The magnitude of the epistatic components tends to be larger when the frequencies of the dominant genes are reduced. With duplicate epistasis (A_B_,

Table 5. Relative magnitudes (\%) of the components of genotypic variance, considering duplicate genes with cumulative effects and populations with high frequencies of dominant genes (p), in different polygenic systems

| $\theta$ | k | k' | p | $\sigma_{A}^{2} / \sigma_{G}^{2}$ | $\sigma_{\mathrm{D}}^{2} / \sigma_{\mathrm{G}}^{2}$ | $\sigma_{\mathbf{A A}}^{2} / \sigma_{\mathbf{G}}^{2}$ | $\left(\sigma_{\text {AD }}^{2}+\sigma_{\mathbf{D A}}^{2}\right) / \sigma_{\mathbf{G}}^{2}$ | $\sigma_{\mathrm{DD}}^{2} / \sigma_{\mathrm{G}}^{2}$ | $\sigma_{\mathrm{I}}^{2} / \sigma_{\mathrm{G}}^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.1 | 2 | 2 | 0.9 | 18.182 | 81.817 | 0.000 | 0.000 | 0.001 | 0.001 |
| 0.1 | 10 | 2 | 0.9 | 18.182 | 81.818 | 0.000 | 0.000 | 0.000 | 0.000 |
| 0.1 | 10 | 5 | 0.9 | 18.181 | 81.816 | 0.000 | 0.001 | 0.002 | 0.003 |
| 0.1 | 10 | 10 | 0.9 | 18.180 | 81.809 | 0.000 | 0.003 | 0.008 | 0.012 |
| 0.1 | 100 | 20 | 0.9 | 18.181 | 81.814 | 0.000 | 0.001 | 0.003 | 0.005 |
| 0.1 | 100 | 50 | 0.9 | 18.176 | 81.792 | 0.001 | 0.009 | 0.021 | 0.032 |
| 0.1 | 100 | 100 | 0.9 | 18.159 | 81.714 | 0.004 | 0.038 | 0.085 | 0.127 |
| 0.1 | 1000 | 200 | 0.9 | 18.173 | 81.776 | 0.002 | 0.015 | 0.034 | 0.051 |
| 0.1 | 1000 | 500 | 0.9 | 18.124 | 81.556 | 0.011 | 0.095 | 0.214 | 0.320 |
| 0.1 | 1000 | 1000 | 0.9 | 17.951 | 80.779 | 0.042 | 0.378 | 0.851 | 1.271 |
| 1 | 2 | 2 | 0.9 | 18.158 | 81.713 | 0.004 | 0.038 | 0.086 | 0.129 |
| 1 | 10 | 2 | 0.9 | 18.177 | 81.797 | 0.001 | 0.008 | 0.017 | 0.026 |
| 1 | 10 | 5 | 0.9 | 18.135 | 81.608 | 0.008 | 0.076 | 0.172 | 0.257 |
| 1 | 10 | 10 | 0.9 | 17.973 | 80.880 | 0.038 | 0.341 | 0.767 | 1.146 |
| 1 | 100 | 20 | 0.9 | 18.093 | 81.420 | 0.016 | 0.145 | 0.326 | 0.487 |
| 1 | 100 | 50 | 0.9 | 17.626 | 79.315 | 0.101 | 0.910 | 2.048 | 3.060 |
| 1 | 100 | 100 | 0.9 | 16.125 | 72.564 | 0.374 | 3.365 | 7.572 | 11.311 |
| 1 | 1000 | 200 | 0.9 | 17.295 | 77.828 | 0.161 | 1.451 | 3.265 | 4.877 |
| 1 | 1000 | 500 | 0.9 | 13.759 | 61.917 | 0.804 | 7.237 | 16.283 | 24.324 |
| 1 | 1000 | 1000 | 0.9 | 7.950 | 35.776 | 1.860 | 16.743 | 37.671 | 56.274 |
| 2 | 2 | 2 | 0.9 | 18.089 | 81.399 | 0.017 | 0.153 | 0.343 | 0.513 |
| 2 | 10 | 2 | 0.9 | 18.163 | 81.734 | 0.003 | 0.031 | 0.069 | 0.103 |
| 2 | 10 | 5 | 0.9 | 17.996 | 80.984 | 0.034 | 0.303 | 0.683 | 1.020 |
| 2 | 10 | 10 | 0.9 | 17.376 | 78.192 | 0.147 | 1.319 | 2.967 | 4.432 |
| 2 | 100 | 20 | 0.9 | 17.833 | 80.247 | 0.063 | 0.571 | 1.286 | 1.921 |
| 2 | 100 | 50 | 0.9 | 16.144 | 72.647 | 0.371 | 3.335 | 7.504 | 11.210 |
| 2 | 100 | 100 | 0.9 | 12.040 | 54.179 | 1.117 | 10.051 | 22.614 | 33.782 |
| 2 | 1000 | 200 | 0.9 | 15.088 | 67.894 | 0.563 | 5.063 | 11.393 | 17.019 |
| 2 | 1000 | 500 | 0.9 | 7.955 | 35.796 | 1.859 | 16.735 | 37.655 | 56.249 |
| 2 | 1000 | 1000 | 0.9 | 2.957 | 13.308 | 2.768 | 24.913 | 56.054 | 83.734 |

$\sigma_{\mathrm{I}}^{2}$ is the epistatic genetic variance; k is the number of genes and $\mathrm{k}^{\prime}$ is the number of genes that interact.
Table 6. Relative magnitudes (\%) of the components of genotypic variance, considering dominant and recessive epistasis, in relation to different polygenic systems and populations

| k | $\mathbf{k}^{\prime}$ | p | $\sigma_{\mathrm{A}}^{2} / \sigma_{\mathrm{G}}^{2}$ | $\sigma_{D}^{2} / \sigma_{G}^{2}$ | $\sigma_{\mathbf{A A}}^{2} / \sigma_{\mathbf{G}}^{2}$ | $\left(\sigma_{\mathbf{A D}}^{2}+\sigma_{\mathbf{D A}}^{2}\right) / \sigma_{\mathbf{G}}^{2}$ | $\sigma_{\mathrm{DD}}^{2} / \sigma_{\mathrm{G}}^{2}$ | $\sigma_{\mathrm{I}}^{2} / \sigma_{\mathrm{G}}^{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 2 | 0.1 | 72.440 | 4.024 | 21.123 | 2.347 | 0.065 | 23.536 |
| 10 | 2 | 0.1 | 89.243 | 4.958 | 5.205 | 0.578 | 0.016 | 5.799 |
| 10 | 5 | 0.1 | 24.904 | 1.384 | 66.158 | 7.351 | 0.204 | 73.713 |
| 10 | 10 | 0.1 | 5.605 | 0.311 | 84.440 | 9.382 | 0.261 | 94.083 |
| 100 | 20 | 0.1 | 11.269 | 0.626 | 79.075 | 8.786 | 0.244 | 88.105 |
| 100 | 50 | 0.1 | 1.842 | 0.102 | 88.006 | 9.778 | 0.272 | 98.056 |
| 100 | 100 | 0.1 | 0.455 | 0.025 | 89.320 | 9.924 | 0.276 | 99.520 |
| 1000 | 200 | 0.1 | 1.113 | 0.062 | 88.696 | 9.855 | 0.274 | 98.825 |
| 1000 | 500 | 0.1 | 0.178 | 0.010 | 89.582 | 9.954 | 0.276 | 99.812 |
| 1000 | 1000 | 0.1 | 0.045 | 0.002 | 89.708 | 9.968 | 0.277 | 99.953 |
| 2 | 2 | 0.5 | 48.485 | 24.242 | 12.121 | 12.121 | 3.030 | 27.273 |
| 10 | 2 | 0.5 | 62.016 | 31.008 | 3.101 | 3.101 | 0.775 | 6.977 |
| 10 | 5 | 0.5 | 44.444 | 22.222 | 14.815 | 14.815 | 3.704 | 33.333 |
| 10 | 10 | 0.5 | 23.188 | 11.594 | 28.986 | 28.986 | 7.246 | 65.217 |
| 100 | 20 | 0.5 | 38.929 | 19.465 | 18.491 | 18.491 | 4.623 | 41.606 |
| 100 | 50 | 0.5 | 12.608 | 6.304 | 36.039 | 36.039 | 9.010 | 81.087 |
| 100 | 100 | 0.5 | 3.722 | 1.861 | 41.963 | 41.963 | 10.491 | 94.417 |
| 1000 | 200 | 0.5 | 8.640 | 4.320 | 38.685 | 38.685 | 9.671 | 87.040 |
| 1000 | 500 | 0.5 | 1.558 | 0.779 | 43.406 | 43.406 | 10.851 | 97.663 |
| 1000 | 1000 | 0.5 | 0.397 | 0.199 | 44.180 | 44.180 | 11.045 | 99.404 |
| 2 | 2 | 0.9 | 17.829 | 80.230 | 0.064 | 0.578 | 1.300 | 1.942 |
| 10 | 2 | 0.9 | 18.110 | 81.495 | 0.013 | 0.117 | 0.264 | 0.394 |
| 10 | 5 | 0.9 | 17.792 | 80.065 | 0.071 | 0.638 | 1.435 | 2.143 |
| 10 | 10 | 0.9 | 16.961 | 76.323 | 0.222 | 1.998 | 4.496 | 6.717 |
| 100 | 20 | 0.9 | 17.742 | 79.838 | 0.080 | 0.720 | 1.620 | 2.420 |
| 100 | 50 | 0.9 | 15.974 | 71.884 | 0.401 | 3.612 | 8.128 | 12.141 |
| 100 | 100 | 0.9 | 11.890 | 53.504 | 1.144 | 10.296 | 23.166 | 34.607 |
| 1000 | 200 | 0.9 | 15.065 | 67.793 | 0.567 | 5.100 | 11.475 | 17.142 |
| 1000 | 500 | 0.9 | 7.993 | 35.967 | 1.853 | 16.673 | 37.515 | 56.040 |
| 1000 | 1000 | 0.9 | 2.993 | 13.468 | 2.762 | 24.855 | 55.923 | 83.539 |

[^3]Table 7. Relative magnitudes (\%) of the components of genotypic variance, considering two genes and different epistasis

| Epistasis |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | ---: | ---: | ---: |

$\sigma_{I}^{2}$ is the epistatic genetic variance; $p$ is the frequency of the dominant gene.

A_bb and aaB_ with one genotypic value and aabb with another value), the additive x additive epistatic variance should be the most important component of the genotypic variance when the frequencies of the dominant genes are greater than 0.5 . With smaller frequencies of the dominant genes, the epistatic variance is minimized and the component of largest value tends to be the additive variance. Also in the case of dominant and recessive epistasis (A_B_, $A_{-} b b$ and aabb with one genotypic value and aaB_ with another value), the additive variance is the most important component, but the dominance variance becomes larger when the frequencies of the dominant genes are high. With duplicate genes with cumulative effects ( $\mathrm{A}_{-} \mathrm{B}_{-}$ with one genotypic value, $A_{-} b b$ and $a a B_{-}$with another
genotypic value and aabb with a third value), assuming $\mathrm{q}=1$, the additive variance tends to be the component of largest magnitude. However, if the frequencies of the dominant genes are high, the additive x additive variance will be the largest component. The values for recessive epistasis (A_B_ with one genotypic value, $A_{\text {_ }}$ bb with another genotypic value and aaB_ and aabb with a third value), dominant epistasis ( $A_{-} B_{-}$and A_bb with one genotypic value, aaB_ with another genotypic value and aabb with a third value) and non-epistasis genic interaction ( $\mathrm{A}_{-} \mathrm{B}_{-}$, $\mathrm{A}_{-} \mathrm{bb}, \mathrm{aaB} \mathrm{B}_{-}$and aabb with different genotypic values) are practically the same as complementary epistasis, duplicate epistasis and duplicate genes with cumulative effects, respectively.

# Importância relativa dos componentes epistáticos da variância genotípica em populações não endógamas 


#### Abstract

RESUMO - Existem abordagens teóricas sobre interação gênica em sistemas poligênicos e metodologias que permitem confirmar sua ocorrência. Entretanto, alguns aspectos relevantes como as importâncias relativas dos componentes epistáticos da variância genotípica requerem investigação. Foram consideradas epistasia complementar, duplicada, recessiva, dominante, dominante e recessiva, genes duplicados com efeitos cumulativos e interação gênica não-epistática. Independente do tipo de epistasia e das freqüências gênicas, quanto mais complexo o sistema poligênico, quanto maior o número de genes que interagem e quanto maior a magnitude dos efeitos epistáticos, relativamente aos desvios a (diferença entre valor genotípico do homozigoto de maior expressão e média dos valores genotípicos dos homozigotos) e d (devido à dominância), maiores as magnitudes dos


componentes epistáticos. Apenas em sistemas genéticos mais simples ou naqueles com predominância de epistasia complementar, recessiva, dominante e recessiva, genes duplicados com efeitos cumulativos ou interação gênica não epistática, e freqüiências elevadas dos genes dominantes, podem ser reduzidos ou desconsiderados os componentes epistáticos.

Palavras-chave: interação gênica, epistasia digênica, parâmetros genéticos, populações não-endógamas.

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[^1]:    $\sigma_{I}^{2}$ is the epistatic genetic variance; $k$ is the number of genes and $k^{\prime}$ is the number of genes that interact; $p$ is the frequency of the dominant genes.

[^2]:    $\sigma_{\mathrm{I}}^{2}$ is the epistatic genetic variance; $k$ is the number of genes and $k^{\prime}$ is the number of genes that interact; $p$ is the frequency of the dominant genes.

[^3]:    $\overline{\sigma_{\mathrm{I}}^{2}}$ is the epistatic genetic variance; $k$ is the number of genes, $k^{\prime}$ is the number of genes that interact and $p$ in the frequency of the dominant gene.

